On computing of VDR rapid filters control system

O obliczeniach systemu sterowania stacją filtrów VDRF

WOJCIECH DĄBROWSKI

DOI 10.36119/15.2020.9.8

A simple and accurate method of solving the set of equations describing Variable Declining Rate Filters in a bank was described, based on the literature, including among the others the earlier papers of the author. Only the mathematical model developed by Di Bernardo was considered, as the most practical from the engineering point of view.
Key words: filtration, variable declining rate, filter plants

Na podstawie znanej literatury, w tym między innymi własnych publikacji autora opisano prostą i dokładną metodę rozwiązania układu równań opisujących działanie stacji filtrów o skokowo zmiennej wydajności. Ta metoda obliczeń odnosi się do modelu matematycznego Luiz Di Bernardo, jako najpraktyczniejszego z inżynierskiego punktu widzenia.
Słowa kluczowe: filtracja, skokowo zmienna wydajność, stacje filtrów.

Introduction

Variable Declining Rate Filters (VDRF) are constructed in the similar way as rapid water constant rate filters, but instead of flow-rate controllers, orifices are installed at outflows from all filter units. An example of this construction is presented in Fig.1.

VDR Filters should fulfill the following requirements:
1. consist of at least four filter units,
2. have inflows to filters located below the lowest water level above filters,
3. head loss in pipes being negligible in comparison with head loss of flow through the filters.

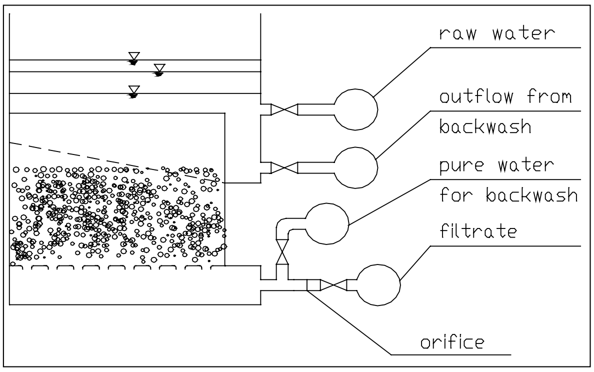


Fig.1
The vertical cross section through a gravitational VDRF filter. The figure is very similar to the one published by W.Dąbrowski, "The progression of flow rates in Variable Declining Rate Filter systems, Acta hydrochimica et hydrobiologica, 2006,Vol.34, Issue 5, 442-452. Copyright Wiley-VCH Verlag GmbH &Co. KGaA. Reproduced with permission.

Filters operated under Variable Declining Rate operation rules produce water approximately of the same quality as received under Constant Rate Operation [14].

The pattern of water table level and flow-rates for such a filter plant is presented in Fig.2. The figure was constructed based on a numerical simulation model of a Variable Declining Rate (VDR) filter plant. Results of computations are denoted by the dotted line, and empirical measurements taken from literature [1], represented by the solid lines. In Fig.2, flow-rate through a filter "i" is denoted by q_i, and a filter surface area by "a".

The first requirement limits the sudden increase in flow-rates through all operating filters at the moment when one unit is disconnected for backwashing [2]. The second and the third requirements ensure that the time-varying water level above each filter may be assumed to be the same above all filter media at any moment of operation [4].

The orifices installed at the outflows from all filters create head losses in conditions of turbulent flow. The operation of a Variable Declining Rate (VDR) filter plant is based on the time-varying interaction between increasing head losses of flow through the filter media, and decreasing head losses of flow through orifices. Proper design requires the limitation of the flow-rate $q_1 = q_{max}$ through a freshly backwashed filter, which results from the properly designed head loss of flow through the orifice. As the flow is laminar-linear through the filter media and turbulent through orifices, the head loss of flow through orifices is much more sensitive to changes in flow-rate than the head loss of flow through filter media [4],[10],[11]. Thus, smaller flow-rates through previously

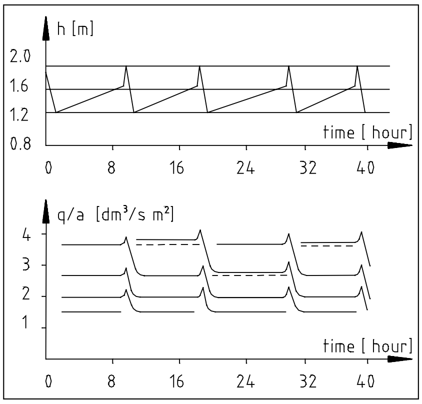


Fig.2
Pattern of water table fluctuations (upper part of the drawing) and flow rate changes in Variable Declining Rate mode according to computations; —— and theory, ------ experimental data for the Medmenhan filter plant [1]. The figure reprinted from Dąbrowski W., "The progression of flow rates in Variable Declining Rate Filter systems, Acta hydrochimica et hydrobiologica, 2006,Vol.34, Issue 5, 442-452. Copyright Wiley-VCH Verlag GmbH &Co. KGaA. Reproduced with permission.

prof. dr hab. inż. Wojciech Dąbrowski, https://orcid.org/0000-0003-0896-793X – Cracow University of Technology, Cracow, Poland.
Corresponding author: wdabrow@pk.edu.pl

backwashed filters result in a sharp decrease of head loss of flow through orifices. Finally the flow-rate through the most recently backwashed filter is restricted mainly by the head loss created by the orifice while the flow-rate through significantly clogged filters depends mostly on the filter media resistance. The most significant changes in flow-rates happen just after returning a backwashed filter to operation [2],[10], so the pattern of water level and flow-rates may be described by Fig.2 and the filters are called "Variable Declining Rate Filters" (VDRF).

Designing method

Di Bernardo made the following assumptions in his mathematical model [11], [12]:

1. flow-rate changes through all operating filters occur mostly just after disconnecting a filter for a backwash, and then when putting it into service again,
2. the period of a backwash is short enough, in comparison with periods between subsequent backwashes in a plant, to consider that there is no change in the resistance of all operating filters before disconnecting and after reconnecting the most clogged filter for backwash,
3. changes of the accumulation of water above filter media due to fluctuations in water surface level are negligible in comparison with water production.

The method of design is described by the assumptions 1,2,3 and Figs. 2,3,. In Fig.3, line 1 represents head loss in clean porous media during laminar linear flow, and line 2 represents head loss of flow through the orifice and drainage, which are turbulent and transitional, respectively. The level P_2 is reached by the water surface just before a subsequent backwashing in a plant, and P_1 just after starting the operation of a freshly backwashed filter. Using the notations illustrated in Fig.3 the head loss of flow through a filter equals H just before and H-h_o just after a backwash in a plant.

The drawing is similar to one of drawings published previously by Di Bernardo [11],[12].

Keeping in mind the assumptions suggested by Di Bernardo [11],[12] and summarized above by points 1, 2, and 3, the mass balance is described by equation (1), and the head loss of flow through a filter by equation (2).

$$Q = \sum_{i=1}^{i=z} q_i \quad 1 \text{ equation} \qquad (1)$$

$$H - h_o = c_1 q_1 + c_2 (q_1)^n \; 1 \text{ equation} \quad (2)$$

In equations (1),(2) the following notations were used:

- c_1 – proportional coefficient characterizing the resistance of a clean filter medium,
- c_2 – coefficient of head losses created by turbulent flow through an orifice and transitional through the drainage,
- H – total head loss of flow through the plant just before a backwash,
- h_o – the height of water surface fluctuations between backwashes,
- i – number of filters in the bank,
- n – exponent of head losses created by flow through the drainage and the orifice,
- q_i – flow-rate through a filter i (i=1… …z, where z is the number of filters in a plant)
- Q – the total inflow to the plant.

The results of calculations based on Di Bernardo's model [11],[12] are so close to experimental data that it has been decided here to ignore minor effect of sediment compressibility impact on head loss to flow through the porous media . The total head loss of flow through all filters is H just before a backwash, and H-h_o after the backwash. Considering a flow-rate q_i just before a subsequent backwash of a filter "z" in a plant, the head loss of flow through the media at that moment can be calculated by subtracting head loss of flow through drainage and orifice $c_2(q_i)^n$ from the total head loss H. After the backwash of the filter "z", the flow rate through the filter "i" rapidly decreases to the value q_{i+1}, and analogically the head loss created by the linear laminar flow through the media of this filter equals to $H - h_o - c_2(q_{i+1})^n$. Following the second assumption of the Di Bernardo model [11], [12], we consider the media resistance of any filter "i" (where i=1 …..i= z-1) to be the same after a backwash of the filter "z" as it was before this backwash. Finally, for a plant consisting of z filters, z-1 equations may be written [6], [7], [8]:

$$\frac{H - h_o - c_2(q_{i+1})^n}{q_{i+1}} = \frac{H - c_2(q_i)^n}{q_i} \quad \text{(z-1 equations)} \qquad (3)$$

Using z-1 equations of type (3) reduces the number of equations describing the original Di Bernardo model [11],[12] by z-1 equations.

As the system of equations (1),(2),(3) consists of z+1 equations, z+1 unknown variables may be calculated from it. Usually Q,c_2, c_1,n,H are known and q_i, h_o are computed. Then the ratio of $q_1/q_{avr} = q_{max}/q_{avr}$ is calculated and verified if it does not exceed the limit of 1.5 established as a rule of thumb in the U.S.A. Too high a ratio of q_{max}/q_{avr} may result in poor filtrate quality. If it exceeds the limit, a different orifice is chosen (c_2,n) and the calculations are repeated. However, from the engineering practice it is well known that sharp changes of flow are much more disturbing the filtrate quality than high stable velocities of filtration.

Despite the fact that h_o is usually an unknown variable, the simplest way of solving the system of equations (1),(2),(3) consists of the following steps :

- assuming a primary value of h_o,
- calculating q_1 from equation (2),
- calculating q_2…….q_z from equation (3) step by step, q_2 from q_1, q_3 from q_2, etc.,
- verifying the mass balance equation (1), and if not satisfied change the previously assumed value of h_o. If the left hand side of equation (1) is higher than the right one, the value of h_o should be decreased in the next iteration. If it is lower the h_o should be increased.

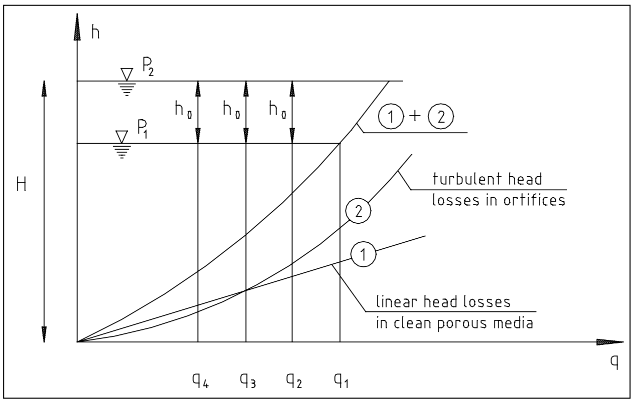


Fig.3
Illustration of the Di Bernardo model and of the notation used in equations (1),(2),(3) – H, h_o,q_i. The figure reprinted from Dąbrowski W., "The progression of flow rates in Variable Declining Rate Filter systems, Acta hydrochimica et hydrobiologica, 2006,Vol.34, Issue 5, 442-452. Copyright Wiley-VCH Verlag GmbH &Co. KGaA. Reproduced with permission.

Computations

Equation (3) may be rewritten again as (3*):

$$\frac{q_{i+1}}{q_i} = \frac{H-h_o}{H + c_2 \cdot q_i^n \cdot [(\frac{q_{i+1}}{q_i})^{n-1} - 1]} \qquad (3^*)$$

Because $(q_{i+1}/q_i)^{n-1} < 1$, so:

$$\frac{q_{i+1}}{q_i} > \frac{H-h_o}{H} \qquad (4^*)$$

which bounds the ratio of q_{i+1}/q_1 from the lower side. This lower bound may be used again to substitute the term $(q_{i+1}/q_i)^{n-1}$ into the denominator of the right hand side of equation (3*), creating this time the upper bound for the ratio q_{i+1}/q_i:

$$\frac{q_{i+1}}{q_i} < \frac{H-h_o}{H + c_2 \cdot q_i^n \cdot [(\frac{H-h_o}{H})^{n-1} - 1]} \qquad (5^*)$$

This procedure may be repeated, each time receiving more tidy upper and lower bounds for q_{i+1}/q_1. Dabrowski W. [7] , [8] used this approach to develop an accurate and fast method of solving the set on non-linear equations (1),(2),(3). The maximum possible error of computations is known and may be easily decreased by the next bounds constructed in the similar way. The error of computations equals zero for the first filter and progresses with the number of filters in the plant. In Fig.4, the value of this error defined as $\{(q_{i+}-q_{i-})q_{i,num}\}\cdot 100\%$ is presented for computations done for the same data known from literature for an existing plant [11],[12]. By q_{i+} and q_i – upper and lower bounds for q_i are denoted, and $q_{i,num}$ denotes the value of q_i computed by a precise numerical method. The accuracy of the solution presented in Fig.4 is very high in comparison with simplifications of the phenomena accompanying the filtration process assumed for formulating the system of equations (1),(2),(3), and there was no reason to seek any more accurate solutions.

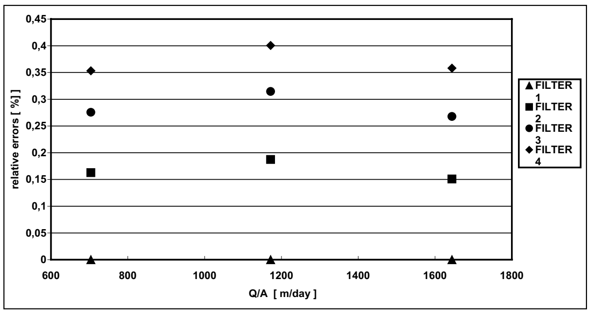


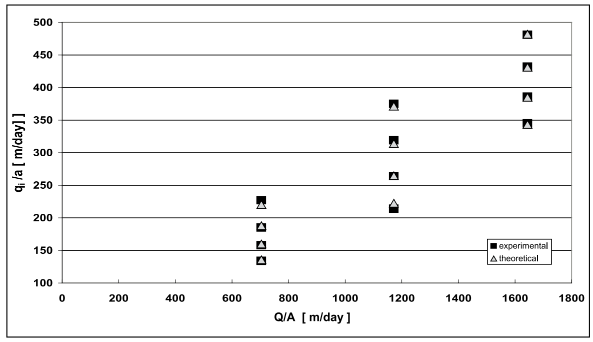
Fig.4
An illustration of the accuracy of the method of solving the system of equations (1),(2),(3). The figure reprinted from Dąbrowski W., "The progression of flow rates in Variable Declining Rate Filter systems, Acta hydrochimica et hydrobiologica, 2006, Vol.34, Issue 5, 442-452. Copyright Wiley-VCH Verlag GmbH &Co. KGaA. Reproduced with permission.

The data for computations are described in the publications [4], [7]. Three different plant capacity values Q are considered. The relative errors are defined as equal to

$$\{(qi+-qi-)qi,num\}\bullet 100\%.$$

A comparison of computed q_i with some measurements reported by Di Bernardo are presented in Fig.5. The simplified here Di Bernardo's model [11],[12] is well suited to the results of measurements [15]. In our laboratory experiments the model has been also verified showing its applicability for technical purposes.

Fig.5
Values of q_i/a measured and computed for one of VDR Filter plants described by Di Bernardo [11],[12]. The figure reprinted from Dąbrowski W., "The progression of flow rates in Variable Declining Rate Filter systems, Acta hydrochimica et hydrobiologica, 2006,Vol.34, Issue 5, 442-452. Copyright Wiley-VCH Verlag GmbH &Co. KGaA. Reproduced with permission.



Primary values of h_o

In a rough approximation it can be predicted that flow-rates are elements of a geometrical progression $q_{i+1}/q_i \approx 1-h_o/$

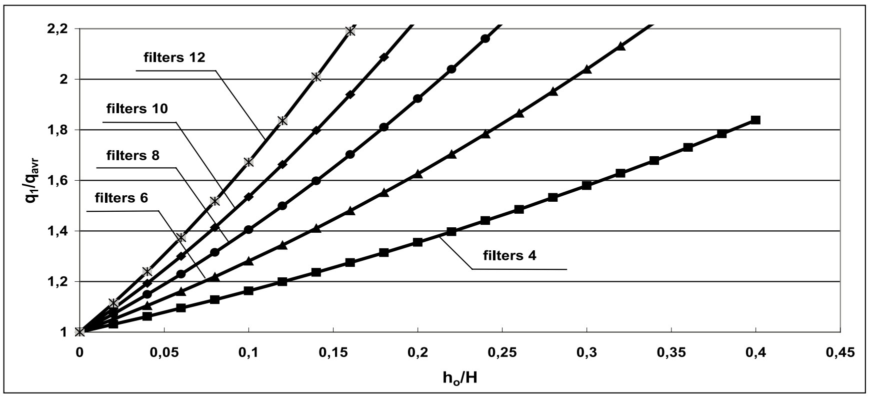


Fig.6
Primary values of ho for the methods of calculation described here. The figure reprinted from Dąbrowski W., "The progression of flow rates in Variable Declining Rate Filter systems, Acta hydrochimica et hydrobiologica, 2006,Vol.34, Issue 5, 442-452. Copyright Wiley-VCH Verlag GmbH &Co. KGaA. Reproduced with permission.

H [7]. From that a monogram for the primary values of h_o was constructed [4], [7].

Large and small filter plants

Describing flow rates through filters by elements of geometrical progression of the constant value lower than one suggests that the total capacity of the plant is bounded even for infinite number of filter units. Because of that it was suspected that large filter plants are less economical than plants consisted of lower number of units [4], [7]. This problem has been investigated in numerical computations [9] based on the system of equations (1), (2), (3). They confirmed an uneconomical operation of plants consisted with many units under similar values of the height of water table fluctuations h_o as used in operating of small filter plants. However, if h_o value is properly chosen, to ensure the required ratio value of q_1/q_{avr} the operation of large plants can be economical. However, the values of h_o for many filters in a bank can be so small to make the operation of the plant difficult.

Conclusions

A theoretical model of VDRF plants developed originally by Di Bernardo [11],[12] has been reduced in the number of equations [7] and has been proven to be quite accurate despite the fact that it does not contain any unknown or difficultly measured parameters. The method of solution to the system of equations governing the model is simple, fast and accurate. It can be also used for unstable raw water quality [13] and temperature [5].

List of main symbols

- a – surface area of a filter,
- A – total surface of all filters in a plant,
- c_1 – proportional coefficient characterizing the resistance of the clean filter medium,
- c_2 – coefficient of turbulent head losses,
- H – total head loss of flow through filters, drainages, and orifices just before a backwash
- h_o – the height of water table increase between the levels P_1 and P_2,
- n – exponent of the turbulent head loss,
- P_1 – water surface level after a backwash,
- P_2 – water surface level before a backwash,
- q_i – flow-rate through i-unit,
- Q – the total flow-rate through the plant,
- t – time,
- z – a number of filters in a bank.

REFERENCES

[1] Arboleda J., Giraldo R., Snel H. 1985. Hydraulic behaviour of declining rate filtration. Journal of the American Water Works Association 77(12): 67-74.

[2] Cleasby,J.L.,(1969): Filtrate rate control without rate controllers. J. Am.Water Works Assoc. **61**(4), 181 – 185.

[3] Dąbrowski W., Consequences of the mass balance simplification in modelling deep bed filtration, Water Research, 1988,22,10, 1219 – 1227

[4] Dąbrowski W. 1994. Investigations into Variable Declining Rate Filters – Hydraulics and Design Parameters. Monograph 170 edited by the Cracow University of Technology (in Polish).

[5] Dąbrowski, W., Mackie, R.I.(1994): Influence of temperature on the performance of variable declining rate filters for drinking water. Arch. Hydro-Eng. Environ. Mechanics 41,3-4, 37-51.

[6] Dąbrowski, W., Mackie, R.I.(1997): Dynamics of Variable Declining Rate Filters during backwash. Arch. Hydro-Eng. Environ. Mechanics. 41(3-4), 37-51.

[7] Dąbrowski W., The progression of flow rates in Variable Declining Rate Filter systems, Acta hydrochimica et hydrobiologica, 2006, Vol.34, Issue 5, 442-452

[8] Dąbrowski W., Rational operation of Variable Declining Rate Filters, Environment Protection Engineering, 2011,37,4, 35-53

[9] Dąbrowski W., Should variable declining rate filters be operatem as one large or several separate plants? Desalination and Water Treatment, 2020, 185, 105-110

[10] Di Bernardo,L., Cleasby J.L., 1980,Declining-rate versus constant –rate filtration, Jour.Environ.Engng.Div.,ASCE,106(EE6),1023-1041

[11] Di Bernardo, L.(1986): A rational method to the design of declining rate filters. 4th World Filtration Congress, held at Ostend, Belgium, Apr. (manuscript).

[12] Di Bernardo, L.(1987): Designing Declining-rate Filters. Filtration & Separation. Sept./Oct., 338 –341.

[13] Mackie, R.I., Dąbrowski, W., Zielina, M.(2003): Numerical study of a rational rule for the operation of variable declining rate filters in response to changes in raw water quality. Environment Protection Engineering. 29,1,45-51 (2003).

[14] Mackie, R.I., Zielina, M., Dąbrowski, W.(2004);Filtrate quality from different filter Operations", Acta hydrochim.hydrobiol.31(1),25-35.

[15] Zielina M., Dąbrowski W., Mackie R.I., Laboratory and numerical experiments into efficient management of VDR filter plants, Environmental Protection Engineering, 2015, 41, 3, 101-120

■